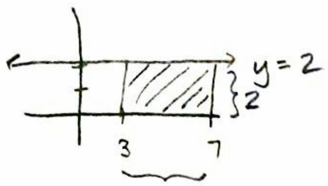
Lesson 30: Definite Integrals

Def $\int_a^b f(x)\,dx$, "the integral from a to b of f(x) with respect to x", is called a definite integral.

$\int_a^b f(x)\,dx$ is the area under f(x) on [a,b].

Ex 1 $\int_3^7 2\,dx = 4(2) = \boxed{8}$

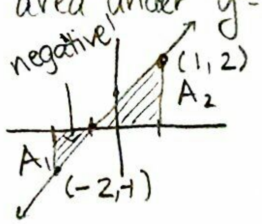
area under $y=2$ on $[3,7]$



$7-3=4$

Ex 2 $\int_{-2}^{1} x+1\,dx = -\frac{1}{2}+2 = \boxed{\frac{3}{2}}$

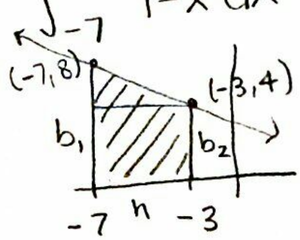
area under $y=x+1$ on $[-2,1]$



$A_1 = \frac{1}{2}(1)(1) = \frac{1}{2}$ ← should be negative

$A_2 = \frac{1}{2}(2)(2) = 2$

Ex 3 $\int_{-7}^{-3} 1-x\,dx$



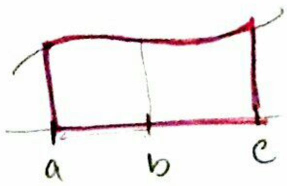
$-3-(-7)=4$

$A_{trap} = \frac{1}{2}(b_1+b_2)h$

$= \frac{1}{2}(8+4)(4) = \frac{1}{2}(12)(4) = \boxed{24}$

Properties of Definite Integrals

- $\int_a^c f(x)dx = \int_a^b f(x)dx + \int_b^c f(x)dx$



- $\int_a^b kf(x)dx = k\int_a^b f(x)dx$
- $\int_a^b f(x)dx \pm \int_a^b g(x)dx = \int_a^b f(x) \pm g(x)dx$
- $\int_b^a f(x)dx = -\int_a^b f(x)dx$

Ex 4 If $\int_0^1 f(x)dx = 2$, $\int_0^1 g(x)dx = 3$, find

$$\int_0^1 4f(x) - 7g(x)dx. = \int_0^1 4f(x)dx - \int_0^1 7g(x)dx$$

$$= 4\int_0^1 f(x)dx - 7\int_0^1 g(x)dx$$

$$= 4(2) - 7(3)$$

$$= \boxed{-13}$$